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Let $h^2(R^2c^2 - l^2r^2) = b^2l^4$, $R^2c/l^2 = d$.

Then $A = (l/h)\sqrt{\{(r^2 - u^2)[b^2 + (u + d)^2]\}}$.

This can be integrated by a process similar to that employed by Professor Finkel in Vol. V, No. 1, pages 20, 21.

Let $Rc - lr$, then $A = \frac{1}{h}(lu + Rr)\sqrt{(r^2 - u^2)} = \frac{R}{rh}(cu + r^2)\sqrt{(r^2 - u^2)}$.

$$\begin{aligned}\therefore V &= \frac{2R}{hr} \int_{-r}^r (cu + r^2)\sqrt{(r^2 - u^2)} du + \frac{2Rr}{3h} \int_{-r}^r \frac{(u + c)^2 du}{\sqrt{(r^2 - u^2)}} \\ &= \frac{2\pi Rr}{3h} (2r^2 + c^2).\end{aligned}$$

111. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

(a). Find the dimensions of a cup, capacity c , in the form of a frustum of a regular pyramid of n faces, so that its internal surface is a minimum.

(b). Find the dimensions of a cup, capacity c , in the form of a frustum of a hyperboloid or of a paraboloid, whichever it is, so that its internal surface is a minimum.

Solution by the PROPOSER.

(a). Let $PO = x$, $PG = y$, $\angle AOB = \angle DGE = 2\pi/n$, $AO = r$, where O is the center of the circle circumscribing the larger base and G the center of the circle circumscribing the smaller base.

Then $DG = ry/x$, $AB = 2r\sin(\pi/n)$, $DE = (2ry/x)\sin(\pi/n)$. Area $DGE = (r^2y^2/x^2)\sin(\pi/n)\cos(\pi/n)$.

\therefore Area of upper base $= (nr^2y^2/x^2)\sin(\pi/n)\cos(\pi/n)$.

$$\begin{aligned}PC &= \sqrt{(PA^2 - AC^2)} = \sqrt{[x^2 + r^2 - r^2\sin^2(\pi/n)]} \\ &= \sqrt{[x^2 + r^2\cos^2(\pi/n)]}.\end{aligned}$$

Similarly $DF = (y/x)\sqrt{[x^2 + r^2\cos^2(\pi/n)]}$.

$$\text{Area } ADEB = r\sin(\pi/n)\sqrt{[x^2 + r^2\cos^2(\pi/n)]}[(x^2 - y^2)/x^2].$$

The surface is the least when the upper or smaller base is the bottom of the cup.

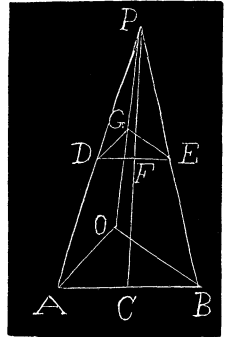
Total surface of cup $= u$, volume $= c$.

$$\therefore u = nr\sin(\pi/n)\sqrt{[x^2 + r^2\cos^2(\pi/n)]}\left(\frac{x^2 - y^2}{x^2}\right) + \frac{nr^2y^2}{x^2}\sin(\pi/n)\cos(\pi/n)$$

$$c = \frac{1}{3}nr^2\sin(\pi/n)\cos(\pi/n)\left(\frac{x^3 - y^3}{x^2}\right). \quad \text{Let } (r/x)\cos(\pi/n) = \tan\theta.$$

$$\therefore u = n\tan\theta\sec\theta\sec(\pi/n)\sin(\pi/n)(x^2 - y^2) + ny^2\tan^2\theta\sec(\pi/n)\sin(\pi/n). \quad (1).$$

$$c = \frac{1}{3}n\tan^2\theta\sec(\pi/n)\sin(\pi/n)(x^3 - y^3) \dots (2).$$



Differentiating (1) and (2) we get

$$\frac{dx}{dy} = \left(\frac{\sec\theta - \tan\theta}{\sec\theta} \right) \frac{y}{x} = (1 - \sin\theta) \frac{y}{x} \dots (3). \quad \frac{dx}{dy} = \frac{y^2}{x^2} \dots (4).$$

$$\frac{dx}{d\theta} = - \frac{(\sec^2\theta + \tan^2\theta)(x^2 - y^2) + 2y^2 \sec\theta \tan\theta}{2x \tan\theta} \dots (5).$$

$$\frac{dx}{d\theta} = - \frac{2\sec^2\theta(x^3 - y^3)}{3x^2 \tan\theta} \dots (6).$$

$$\text{From (3) and (4) } y = x(1 - \sin\theta) \dots (7).$$

$$\text{From (5) and (6), } \sec^2\theta + \tan^2\theta(x^2 - y^2) + 2y^2 \sec\theta \tan\theta = \frac{4\sec^2\theta(x^3 - y^3)}{3x}$$

$$\text{This reduces to } (1 + \sin^2\theta)(x^2 - y^2) + 2y^2 \sin\theta = 4(x^3 - y^3)/3x \dots (8).$$

$$(7) \text{ in } (8) \text{ gives } (3 - 8\sin\theta + 3\sin^2\theta)\sin\theta = 0.$$

$$\therefore \sin\theta = 0 \text{ or } \frac{1}{3}(4 \pm \sqrt{7}).$$

$\sin\theta$ cannot be zero nor greater than unity.

$$\therefore \sin\theta = \frac{1}{3}(4 - \sqrt{7}) \dots (9).$$

$$\therefore \tan\theta = \sqrt{\frac{4\sqrt{7}-7}{14}} \dots (10).$$

$$\text{From (7) and (9), } 3y = x(\sqrt{7} - 1) \dots (11).$$

$$(10) \text{ and } (11) \text{ in (2) gives } x = \frac{1}{3} \left(\frac{2(38\sqrt{7} + 89)c}{n \tan(\pi/n)} \right)^{\frac{1}{3}}.$$

$$\text{From (11), } y = \frac{1}{3}x(\sqrt{7} - 1) = \frac{1}{3} \left(\frac{4(\sqrt{7} + 13)c}{n \tan(\pi/n)} \right)^{\frac{1}{3}}.$$

$$x - y = x - \frac{1}{3}x(\sqrt{7} - 1) = \frac{1}{3}x(4 - \sqrt{7}) = \left(\frac{2(\sqrt{7} - 2)c}{n \tan(\pi/n)} \right)^{\frac{1}{3}} = \text{altitude.}$$

$$r = x \tan\theta \sec(\pi/n) = \left(\frac{(91 + 88\sqrt{7})c^2}{98n^2 \tan^2(\pi/n)} \right)^{\frac{1}{3}} \sec(\pi/n).$$

$$AB = 2r \sin(\pi/n) = 2 \left(\frac{(91 + 88\sqrt{7})c^2}{98n^2 \tan^2(\pi/n)} \right)^{\frac{1}{3}} \tan(\pi/n), \text{ side lower base.}$$

$$DE = \frac{2ry}{x} \sin(\pi/n) = 2 \left(\frac{4(11\sqrt{7} - 28)c^2}{49n^2 \tan^2(\pi/n)} \right)^{\frac{1}{3}} \tan(\pi/n), \text{ side upper base.}$$

(2). Let $y^2 = 4ax$ be the equation to the parabola, then we get from the Integral Calculus the two equations, between the limits x_2 and x_1 ,

$$u = \frac{8}{3}\pi \sqrt{a} [(x_2 + a)^{\frac{3}{2}} - (x_1 + a)^{\frac{3}{2}}] + 4\pi ax_1 \dots (1).$$

$$c = 2\pi a(x_2^2 - x_1^2) \dots (2).$$

From (1) and (2), by differentiation, we get

$$\frac{dx_2}{dx_1} = \frac{(x_1 + a)^{\frac{1}{2}} - \sqrt{a}}{(x_2 + a)^{\frac{1}{2}}} \dots (3); \quad \frac{dx_2}{dx_1} = \frac{x_1}{x_2} \dots (4).$$

$$\frac{dx_2}{da} = - \frac{(x_2 + a)^{\frac{3}{2}} - (x_1 + a)^{\frac{3}{2}} + 3a(x_2 + a)^{\frac{1}{2}} - 3a(x_1 + a)^{\frac{1}{2}} + 3\sqrt{a} x_1}{3a(x_2 + a)^{\frac{1}{2}}} \dots (5).$$

$$\frac{dx_2}{da} = - \frac{x_2^2 - x_1^2}{2ax_2} \dots (6).$$

From (3) and (4) by eliminating dx_2/dx_1 we get

$$x_1 = \frac{x_2^2 - 2x_2\sqrt{a(x_2 + a)}}{x_2 + a} \text{ and } x_1 = 0 \dots (7).$$

Eliminating dx_2/da between (5) and (6) and substituting the first value of x_1 from (7) in the resulting equation, we get after reduction

$$36x_2^3 - 100ax_2^2 - 279a^2x_2 - 144a^3 = 0 \dots (8).$$

Let $a/x_2 = z$ and (8) becomes

$$144z^3 + 279z^2 + 100z - 36 = 0 \dots (9).$$

Let $z = v - \frac{31}{48}$ and (9) becomes

$$v^3 - \frac{1283}{2304}v - \frac{8833}{56256} = 0 \dots (10).$$

$$\therefore v_1 = .861506, z_1 = .215673.$$

$$v_2 = -.445379, z_2 = 1.094212.$$

$$v_3 = -.416138, z_3 = -1.061971.$$

$$\therefore a = .215673x_2, -1.094212x_2, \text{ or } -1.061971x_2.$$

a cannot be negative. The first value of a gives $x_1 = -.019813x_2$, a negative value and therefore not admissible. From this we learn that the second value of x_1 in (7), $x_1 = 0$ is the possible value and the cup is not a frustum of a paraboloid, but a paraboloid, a cup with a curved bottom.

$$x_1 = 0 \text{ in (5) and (6) } x_2^2 - 15ax_2 + 48a^2 = 0.$$

$$\therefore x_2 = \frac{15 \pm \sqrt{33}}{2} a. \quad \therefore x_2 = \frac{15 - \sqrt{33}}{2} a \dots (11) \text{ is the admissible value.}$$

$$\text{From (11) and (2), } x_2 = \left(\frac{(15 - \sqrt{33})c}{4\pi} \right)^{\frac{1}{2}} = \text{altitude of cup.}$$

$$a = \left(\frac{(43 + 5\sqrt{33})c}{3072\pi} \right)^{\frac{1}{2}} = \left(\frac{2c}{\pi(15 - \sqrt{33})^2} \right)^{\frac{1}{2}}.$$

$$y_2 = 2\sqrt{ax_2} = 2 \left(\frac{(15 + \sqrt{33})c^2}{384\pi^2} \right)^{\frac{1}{2}} \text{ radius of top.}$$

$$u = \frac{1}{9} \left(\frac{(1337 + 215\sqrt{33})4\pi c^2}{9} \right)^{\frac{1}{2}} [(3298 - 450\sqrt{33})^{\frac{1}{2}} - 2].$$

MECHANICS.

116. Proposed by C. L. CHILTON, Greensboro, Ala.

Given, the shaft ABC attached at one end by a pivot to the piston-rod of an engine (at A) and the other to a crank of the wheel CDE (at C). The shaft moves through the distance of two feet in one second from A to B and at the same time turns the crank from C to E . The force propelling the shaft along the constrained course from A to B is 5760 pounds. The mass of the rod and wheel and friction being not considered, what would be the kinetic energy of the machine? or the sum of the moment around O , the center of the wheel?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let AC , the connecting rod $= l$, $OC = r =$ one foot, $\angle AOC = \theta$, force 5760 pounds along $AO = P$, component of P along $AC = Q$. Then moment of crank effect about $O = Q \cdot OM$. In the right triangles AOM , AON , $AO:OM = AN:ON$.

$$\therefore P:OM = Q:ON.$$

$$\therefore Q \cdot OM = P \cdot ON.$$

$$\text{Also } ON:OC = \sin OCN:\sin ONC.$$

$$\text{Let } \angle OAC = \varphi. \text{ Then } \angle ONC = \frac{1}{2}\pi - \varphi, \angle OCN = \theta + \varphi.$$

$$\therefore ON = \frac{r \sin(\theta + \varphi)}{\cos \varphi} = r \sin \theta + \frac{r \cos \theta \sin \varphi}{\cos \varphi}.$$

$$\text{but } \sin \varphi = \frac{r \sin \theta}{l}. \therefore ON = r \sin \theta + \frac{r^2 \sin \theta \cos \theta}{\sqrt{(l^2 - r^2 \sin^2 \theta)}}.$$

$$\therefore \text{moment} = Pr \sin \theta \left(1 + \frac{r \cos \theta}{\sqrt{(l^2 - r^2 \sin^2 \theta)}} \right).$$

Now θ varies from 0 to π .

$$\therefore \text{Average moment} = \frac{Pr \int_0^\pi \sin \theta \left(1 + \frac{r \cos \theta}{\sqrt{(l^2 - r^2 \sin^2 \theta)}} \right) d\theta}{\int_0^\pi d\theta} = \frac{2Pr}{\pi},$$

a result independent of the connecting rod.

$$2Pr/\pi = .6366Pr = 3666.816 \text{ or } 3667 \text{ pounds.}$$

117. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pa.

How much lower must *one end* of a heavy uniform chain, wound round the circumference of a perfectly rough vertical wheel, hang than *the other end*, when the chain is on the point of motion?

